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The singular seesaw mechanism with hierarchical Dirac neutrino mass

Y. Chikira[1,a], N. Haba[2,b], Y. Mimura[3,c]

[1] Department of Physics, Tokyo Institute of Technology Oh-okayama, Meguro, Tokyo 152-0033, Japan
[2] Faculty of Engineering, Mie University, Mie, 514-0008, Japan
[3] Theory Group, KEK, Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan

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Abstract. The singular seesaw mechanism can naturally explain the atmospheric neutrino deficit by maximal oscillations between $\nu_{\mu_{\rm L}}$ and $\nu_{\mu_{\rm R}}$. This mechanism can also induce three different scales of the neutrino mass squared differences, which can explain the neutrino deficits of three independent experiments (solar, atmospheric, and LSND) by neutrino oscillations. In this paper we show that realistic mixing angles among the neutrinos can be obtained by introducing a hierarchy in the Dirac neutrino mass. In the case where the Majorana neutrino mass matrix has rank 2, the solar neutrino deficit is explained by vacuum oscillations between ν_e and ν_τ. We also consider the case where the Majorana neutrino mass matrix has rank 1. In this case, the matter enhanced Mikheyev–Smirnov–Wolfenstein solar neutrino solution is preferred as the solution of the solar neutrino deficit.

1 Introduction

According to the recent Super-Kamiokande experiment [1], the atmospheric neutrino data indicate oscillations between ν_μ and ν_τ or sterile neutrinos with the maximal mixing

$$\sin^2 2\theta_{\mu x} \sim 1, \tag{1}$$

where x represents τ or sterile neutrinos. The neutrino mass squared difference $\Delta m^2_{\rm atm}$ is of the order of $10^{-3}\,{\rm eV}^2$.

It is well known that two other independent experiments also imply neutrino oscillations. One is the solar neutrino experiment. This experiment implies oscillations between ν_e and other neutrinos, and there are three possible solutions, namely, the large or small mixing angle Mikheyev–Smirnov–Wolfenstein (MSW) solution [2], and the vacuum oscillation solution [3]. The small (large) angle MSW solution suggests [4]

$$\sin^2 2\theta_{ex} \sim (0.4\text{–}1) \times 10^{-2}, \tag{2}$$

with mass squared difference of order $10^{-5}\,{\rm eV}^2$, and the vacuum oscillation solution suggests

$$\sin^2 2\theta_{ex} \sim 0.75\text{–}1, \tag{3}$$

with the mass squared difference of the order of $10^{-10}\,{\rm eV}^2$. The vacuum oscillation solution is now the most suitable solution as regards the electron energy spectrum of the recent Super-Kamiokande experimental data [5], although the small angle MSW solution has been regarded as the most realistic candidate. On the other hand, the LSND experiment measures oscillations between $\bar\nu_\mu$ and $\bar\nu_e$ [6] with a short base line experiment. Although the confirmation of the LSND results awaits future experiments[1], these results indicate a small mixing angle with

$$\sin^2 2\theta_{\mu e} \sim 10^{-2}, \tag{4}$$

with mass squared difference $\Delta m^2_{\rm LSND} \sim 1\,{\rm eV}^2$.

The most interesting mechanism which can naturally explain the smallness of the neutrino masses is the so-called seesaw mechanism [8]. The general mass matrix of neutrinos above $SU(2)_{\rm L}$ breaking is given by

$$\mathcal{M} = \begin{pmatrix} 0 & m_{\rm D} \\ m_{\rm D}^{\rm T} & M_{\rm R} \end{pmatrix}, \tag{5}$$

where $m_{\rm D}$ and $M_{\rm R}$ represent Dirac and Majorana 3×3 flavor space mass matrices, respectively. In the case of $m_{\rm D} \ll M_{\rm R}$, there appear three light neutrinos with mass matrix

$$\mathcal{M}_{\rm light} = -m_{\rm D} M_{\rm R}^{-1} m_{\rm D}^{\rm T}. \tag{6}$$

This is the essence of the seesaw mechanism. It is worth noting that here it is assumed that there exists an inverse matrix $M_{\rm R}^{-1}$, that is, $\det M_{\rm R} \neq 0$. The singular seesaw

[a] e-mail: ychikira@th.phys.titech.ac.jp
[b] e-mail: haba@phen.mie-u.ac.jp
[c] e-mail: mimura@theory.kek.jp

[1] Recent measurements in the KARMEN detector exclude part of the LSND allowed region [7]

mechanism [9,10], which is also called "partially broken seesaw mechanism" [11][2] is just the case of $\det M_{\mathrm{R}} = 0$. Then, some light right-handed neutrinos are not integrated out, and behave as sterile neutrinos. It turns out that mixings between the surviving sterile neutrinos and active neutrinos are large in general because of the pseudo-Dirac context [12]. We can use this mechanism to explain the large mixing of the atmospheric neutrino experiment. If nature adopts four (or more) neutrino oscillations, the singular seesaw mechanism supplies one of the most attractive models.

The authors of [9] discussed this singular seesaw mechanism in the case that there is no hierarchy in the Dirac mass matrix m_{D} and the Majorana mass matrix M_{R}. They did not take the small mixing of the LSND into account. In this paper, we study the singular seesaw mechanism by introducing the hierarchy in the Dirac mass matrix m_{D} in order to explain the small mixing of the LSND experiment.

We will also study whether the hierarchical Dirac mass can induce not only the small mixing of the LSND experiment but also the small mixing of the MSW solar neutrino solution.

This paper is organized as follows: In Sect. 2, we will review the singular seesaw mechanism briefly. In Sect. 3, we introduce the hierarchical Dirac mass matrix, and determine the order of the parameters. We show that the vacuum oscillation solution is preferred as the solution of the solar neutrino deficit in the case where the Majorana neutrino mass matrix has rank 2 and that the MSW solution is preferred in the case where the Majorana neutrino mass matrix has rank 1. In Sect. 4, we give a summary and a discussion.

2 Singular seesaw mechanism

At first, we explain the pseudo-Dirac mass context [12]. In the case of one generation the neutrino mass term above $SU(2)_{\mathrm{L}}$ breaking is given by

$$-\mathcal{L} = \frac{1}{2}\begin{pmatrix} \nu \ \nu^{\mathrm{C}} \end{pmatrix}\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}\begin{pmatrix} \nu \\ \nu^{\mathrm{C}} \end{pmatrix}, \tag{7}$$

where ν and ν^{C} represent (two component) left- and right-handed neutrinos, respectively.

Here we consider the case of $M \ll m$. In this case the mass matrix (7) realizes a large mixing angle of $\sin^2 2\theta = m^2/(m^2 + M^2/4) \sim 1$ between ν and ν^{C}. The eigenvalues of this mass matrix are $\pm m + M/2$, and the neutrino mass squared difference is $\Delta m^2 = 2mM$ [12]. This mass term is almost Dirac but not exact, so it is called the pseudo-Dirac context, which can naturally induce the maximal mixing. The mass term in the opposite case of $M \gg m$ is that of the ordinary seesaw mechanism.

Now let us take three generations into consideration. We take m and M as 3×3 matrices m_{D} and M_{M}, respectively, in (7). The right-handed Majorana neutrino mass matrix M_{M} is assumed to be rank 2 (or 1)[3]. In this case two (one) neutrinos become light by the ordinary seesaw mechanism, and the remaining one (two) neutrino has the pseudo-Dirac mass context. For example, in the rank-2 case, we can obtain the eigenvalues of four light neutrinos [9,10] as

$$\beta m, \quad \beta m, \quad \text{and} \pm m + \beta m, \tag{8}$$

where $\beta = m/M$, in the case of no hierarchy in the mass matrices m_{D} and M_{M}. It is interesting that the two lighter neutrinos' masses and the mass splitting for the pseudo-Dirac neutrinos are on the same scale. Then the three mass squared differences form a geometric series:

$$\Delta m^2 = \beta^2 m^2, \quad \beta m^2, \quad \text{and}\, m^2, \tag{9}$$

and are favorable to explain three known neutrino oscillation modes, namely, solar neutrinos (MSW solution), atmospheric neutrinos and LSND [10]. Furthermore, since the middle scale of the mass squared difference for atmospheric neutrinos corresponds to pseudo-Dirac neutrinos, its maximal mixing is realized naturally.

However, they cannot explain the small mixing angle of LSND nor the small angle MSW solution if there is no mass hierarchy in m_{D} and M_{M}. We will see that the hierarchical Dirac mass matrices lead to a different series of the mass squared difference, which is suitable for the vacuum oscillation solution for the solar neutrino deficit rather than MSW solutions.

The flaw of the singular seesaw mechanism is that the Dirac neutrino mass m needs to be too small (about 1 eV), and it lacks the motivation of the original seesaw mechanism. When we incorporate the singular seesaw mechanism into phenomenological models, we need some extra mechanism to apply the small Dirac neutrino mass. Its smallness will be realized, for example, by the nonrenormalizable interactions [13], though we do not go into details in this paper.

3 Singular seesaw mechanism with hierarchical Dirac neutrino mass matrix

We introduce the hierarchy in the Dirac neutrino mass matrix as

$$m_{\mathrm{D}} = \begin{pmatrix} \epsilon' m_{11} & \epsilon' m_{12} & \epsilon' m_{13} \\ \epsilon m_{21} & \epsilon m_{22} & \epsilon m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}. \tag{10}$$

We can take the hierarchical parameter ϵ and ϵ' as $\epsilon' \leq \epsilon < 1$, when we do not order the left-handed indices by naming of the neutrino flavors. In this paper, we do not consider the hierarchical structure with respect to right-handed indices. The mass term of neutrinos is given by

$$-\mathcal{L} = m_{\mathrm{D}ij}\nu_i\nu_j^{\mathrm{C}} + \frac{1}{2}M_{\mathrm{M}ij}\nu_i^{\mathrm{C}}\nu_j^{\mathrm{C}}. \tag{11}$$

[2] In this paper we call this mechanism "the singular seesaw mechanism"

[3] It is to be explained in the paper we have in preparation how to make the rank of Majorana mass matrix 1 or 2, so we do not discuss this here

We analyze this model in two cases: the rank of the Majorana mass matrix $M_{\rm M}$ is 1 or 2.

At first, we study the case where the Majorana mass $M_{\rm M}$ has rank 2 with $M_{\rm M} = \mathrm{diag}(M_1, M_2, 0)$. After integrating out the heavy neutrinos[4], light neutrinos have masses given by

$$-\mathcal{L} = -\frac{1}{2}\left(\frac{m_{{\rm D}i1}m_{{\rm D}j1}}{M_1} + \frac{m_{{\rm D}i2}m_{{\rm D}j2}}{M_2}\right)\nu_i\nu_j + m_{{\rm D}i3}\nu_i\nu_3^{\rm C}. \tag{12}$$

The mass matrix for $(\nu_1, \nu_2, \nu_3, \nu_3^{\rm C}) \equiv (\alpha, \beta, \gamma, s)$ is given by

$$\mathcal{M} \sim \begin{pmatrix} -\epsilon'^2\beta & -\epsilon\epsilon'\beta & -\epsilon'\beta & \epsilon' \\ -\epsilon\epsilon'\beta & -\epsilon^2\beta & -\epsilon\beta & \epsilon \\ -\epsilon'\beta & -\epsilon\beta & -\beta & 1 \\ \epsilon' & \epsilon & 1 & 0 \end{pmatrix} m. \tag{13}$$

This matrix is diagonalized as

$$U^\dagger \mathcal{M} U \sim \mathrm{diag}(\epsilon'^2\beta m, \epsilon^2\beta m, (1-\beta)m, -(1+\beta)m), \tag{14}$$

where

$$U \sim \begin{pmatrix} 1 & -\epsilon'/\epsilon & \epsilon' & -\epsilon' \\ \epsilon'/\epsilon & 1 & \epsilon & -\epsilon \\ -\epsilon' & -\epsilon & 1 & -1 \\ \epsilon'\beta & -\epsilon\beta & 1 & 1 \end{pmatrix}. \tag{15}$$

Now we estimate the probability of the neutrino oscillations, which is given by

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\sin^2\frac{\Delta m^2_{ij}}{4E}L, \tag{16}$$

where we neglect the CP phase for simplicity. The oscillation amplitude between α and β is given by

$$-4\sum_{i<j} U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}. \tag{17}$$

From (14) we can obtain three scales of mass squared differences of $\Delta m^2_{12} \sim \epsilon^4\beta^2 m^2$, $\Delta m^2_{34} \sim \beta m^2$, and $\Delta m^2_{13} \sim \Delta m^2_{14} \sim \Delta m^2_{23} \sim \Delta m^2_{24} \sim m^2$. We list the amplitudes corresponding to these three oscillations in Table 1. The oscillations between $\gamma \leftrightarrow s$ give a large mixing, which is expected to correspond to atmospheric neutrino oscillations. Then, we fix

$$\beta m^2 \sim 10^{-3}\,\mathrm{eV}^2. \tag{18}$$

The oscillations with $\Delta m^2 \sim m^2$ may correspond to the LSND data, so we fix

$$m^2 \sim 1\,\mathrm{eV}^2. \tag{19}$$

[4] This heavy neutrino mass M turns out to be 1 keV–1 MeV. We integrate out the heavy neutrinos here though the scale is lower than the momentum scales of the neutrino experiments (e.g., about 1 GeV for atmospheric neutrinos). This method of integrating out is used simply because we wish to display our results clearly. Since the heavy neutrinos have very small mixing with light neutrinos, there is considerable validity to our results below

Table 1. Oscillation amplitudes in the case of rank 2

	$\Delta m^2 \sim \epsilon^4\beta^2 m^2$	$\Delta m^2 \sim \beta m^2$	$\Delta m^2 \sim m^2$
$\alpha \leftrightarrow \beta$	$(\epsilon'/\epsilon)^2$	$\epsilon^2\epsilon'^2$	ϵ'^2
$\alpha \leftrightarrow \gamma$	ϵ'^2	ϵ'^2	ϵ'^2
$\alpha \leftrightarrow s$	$\epsilon'^2\beta^2$	ϵ'^2	$\epsilon'^2\beta$
$\beta \leftrightarrow \gamma$	ϵ'^2	ϵ^2	ϵ^2
$\beta \leftrightarrow s$	$\epsilon'^2\beta^2$	ϵ^2	$\epsilon^2\beta$
$\gamma \leftrightarrow s$	$\epsilon'^2\epsilon^2\beta^2$	1	$\epsilon^2\beta$

Then there remain two patterns defined by whether (α, β) is assigned as (e, τ) or as (τ, e). Let us consider both possibilities here.

(1-1) In the case of $(\alpha, \beta) = (e, \tau)$, ϵ' must be of the order of 10^{-1} based on the small mixing of the LSND data. Then the oscillations with $\Delta m^2 \sim \epsilon^4\beta^2 m^2 \sim 10^{-6}\epsilon^4\mathrm{eV}^2$ should correspond to the solar neutrino oscillations.

(i): For the mass squared difference of the MSW solution, we must choose the parameter ϵ to be close to 1. Then it turns out that the ν_μ–ν_τ mixing is large with $\Delta m^2 \sim 1\,\mathrm{eV}^2$. This type of oscillation leads to a contradiction with the atmospheric neutrino data. Therefore we cannot obtain the MSW solution in this pattern.

(ii): On the other hand, the vacuum oscillation solution can be realized when $\epsilon = O(10^{-1})$. We can realize the large mixing of (3) because the corresponding mixing angle is of the order of $(\epsilon'/\epsilon)^2$.

(1-2) In the case of $(\alpha, \beta) = (\tau, e)$, ϵ must be of order 10^{-1} based on the small mixing of the LSND data. In this pattern, the mass squared difference corresponding to the solar neutrino oscillations should be of the order of $10^{-10}\,\mathrm{eV}^2$. Therefore, only the vacuum oscillation solution can be allowed. To explain the large mixing of the solution, ϵ' must satisfy $\epsilon \sim 10^{-1}$. This is the same parameters as the case of (ii) in (1-1).

Next, let us consider the case where the Majorana mass $M_{\rm M}$ has rank 1 as in $M_{\rm M} = \mathrm{diag}(M, 0, 0)$. The mass term of the neutrinos is given by

$$-\mathcal{L} = m_{{\rm D}ij}\nu_i\nu_j^{\rm C} + \frac{1}{2}M\nu_1^{\rm C}\nu_1^{\rm C}. \tag{20}$$

After integrating out $\nu_1^{\rm C}$, the light neutrinos $(\nu_1, \nu_2, \nu_3, \nu_2^{\rm C}, \nu_3^{\rm C}) \equiv (\alpha, \beta, \gamma, s_1, s_2)$ have masses:

$$\mathcal{M} \sim \begin{pmatrix} -\epsilon'^2\beta & -\epsilon'\epsilon\beta & -\epsilon'\beta & \epsilon' & \epsilon' \\ -\epsilon\epsilon'\beta & -\epsilon^2\beta & -\epsilon\beta & \epsilon & \epsilon \\ -\epsilon'\beta & -\epsilon\beta & -\beta & 1 & 1 \\ \epsilon' & \epsilon & 1 & 0 & 0 \\ \epsilon' & \epsilon & 1 & 0 & 0 \end{pmatrix} m. \tag{21}$$

This matrix can be diagonalized as

$$U^\dagger \mathcal{M} U \sim \mathrm{diag}(\epsilon'^2\beta m, (\epsilon - \epsilon^2\beta)m, -(\epsilon + \epsilon^2\beta)m, (1-\beta)m, -(1+\beta)m), \tag{22}$$

Table 2. Oscillation amplitudes in the case of rank 1

	$\Delta m^2 \sim \epsilon^3\beta m^2$	$\Delta m^2 \sim \beta m^2$	$\Delta m^2 \sim \epsilon^2 m^2$	$\Delta m^2 \sim m^2$
$\alpha \leftrightarrow \beta$	$(\epsilon'/\epsilon)^2$	$\epsilon^2\epsilon'^2$	$(\epsilon'/\epsilon)^2$	ϵ'^2
$\alpha \leftrightarrow \gamma$	ϵ'^2	ϵ'^2	ϵ'^2	ϵ'^2
$\alpha \leftrightarrow s_1, s_2$	$(\epsilon'/\epsilon)^2$	ϵ'^2	$\epsilon'^2/\epsilon\beta$	ϵ'^2/ϵ
$\beta \leftrightarrow \gamma$	ϵ^2	ϵ^2	ϵ'^2	ϵ^2
$\beta \leftrightarrow s_1, s_2$	1	ϵ^2	$\epsilon'^2/\epsilon\beta$	ϵ
$\gamma \leftrightarrow s_1, s_2$	ϵ^2	1	$\epsilon\epsilon'\beta$	ϵ

where

$$U \sim \begin{pmatrix} 1 & -\epsilon'/\epsilon & -\epsilon'/\epsilon & \epsilon' & \epsilon' \\ \epsilon'/\epsilon & 1 & 1 & \epsilon & \epsilon \\ \epsilon' & -\epsilon & -\epsilon & 1 & 1 \\ \epsilon'\beta & -1 & 1 & 1 & -1 \\ \epsilon'\beta & 1 & -1 & 1 & -1 \end{pmatrix}. \quad (23)$$

There are four scales of the mass squared differences: $\Delta m^2 \sim \epsilon^3\beta m^2$, βm^2, $\epsilon^2 m^2$ and m^2. The oscillation amplitudes corresponding to these oscillation modes are listed in Table 2. The atmospheric neutrino oscillations can be regarded as $\gamma \leftrightarrow s_1, s_2$. Therefore, the mass squared difference βm^2 must be of the order of $10^{-3}\,\mathrm{eV}^2$. Then, the mass squared difference corresponding to the solar neutrino oscillations should be $\epsilon^3\beta m^2\,\mathrm{eV}^2 \sim 10^{-3}\epsilon^3\,\mathrm{eV}^2$. There are two candidates for the mass squared differences of LSND, namely, $\Delta m^2_{\mathrm{LSND}} \sim \epsilon^2 m^2$ or $\Delta m^2_{\mathrm{LSND}} \sim m^2$. Here we consider both possibilities.

(2-1) In the case of $(\alpha, \beta) = (e, \tau)$, ϵ' must be of the order of 10^{-1} based on the small mixing of the LSND data. As for the solar neutrinos, the vacuum oscillation solution is excluded because the parameter ϵ cannot be smaller than $\epsilon' \sim 10^{-1}$. Then we consider the parameter $\epsilon = O(10^{-1})$ in order to obtain the mass squared difference suitable for the MSW solution. In this case ϵ'/ϵ tends to become close to 1, and in this case, the $\nu_e \leftrightarrow \nu_\tau$ oscillations with $\Delta m^2 \sim \epsilon^2 m^2$ are associated with a large mixing. However, a large mixing of the order of $(\epsilon'/\epsilon)^2 \sim 1$ with $\Delta m^2 > 10^{-3}\,\mathrm{eV}^2$ is excluded by the CHOOZ experiment [14], and we should choose the mixing $(\epsilon'/\epsilon)^2$ to be smaller than $O(10^{-1})$. This choice of parameters leads from the solar neutrino problem to the small angle MSW solution. Therefore, $(\epsilon'/\epsilon)^2$ appears to be of order 10^{-2}. In order to get such a $(\epsilon'/\epsilon)^2$, we need a delicate tuning of the parameters.

(2-2) In the case of $(\alpha, \beta) = (\tau, e)$, $\epsilon'(\epsilon)$ must be of the order of 10^{-1} from a small mixing amplitude of LSND with $\Delta m^2_{\mathrm{LSND}} \sim \epsilon^2 m^2$ ($\Delta m^2_{\mathrm{LSND}} \sim m^2$). For the same reason as (2-1), the vacuum oscillation solution is excluded, and the parameter ϵ should be chosen to be of the order of 10^{-1} for the MSW solution. Though the large angle MSW solution through the $\nu_e \leftrightarrow \nu_s$ oscillation mode seems to be possible, it is not allowed at the 99% C.L. [15] in a two flavor analysis. Therefore, in this case, we cannot help but consider another oscillation mode, namely, $\nu_e \leftrightarrow \nu_\tau$, as the solution for the solar neutrino deficit. As we mentioned in (2-1), the small angle MSW solution through $\nu_e \leftrightarrow \nu_\tau$ oscillation mode seems to be possible. However, since the mixing of ν_e and ν_s is large, we need a detailed analysis of three generation mixing in this case.

4 Conclusion

The recent atmospheric neutrino data of Super-Kamiokande suggests a maximal mixing between ν_μ and other neutrinos. The singular seesaw mechanism is one of the most interesting scenarios that can naturally explain this large mixing angle between $\nu_{\mu_{\mathrm{L}}}$ and $\nu_{\mu_{\mathrm{R}}}$. This mechanism can also induce three independent mass squared differences, which are suitable for the solutions of the solar and atmospheric neutrino anomalies, and the LSND data. The original scenario in [9] cannot explain the small mixing angle of the LSND data nor the small angle solution of MSW. Thus, we introduced a hierarchy in the Dirac neutrino mass matrix, and re-analyzed the singular seesaw mechanism. As a result, we obtain the small mixing solutions of the LSND and MSW as follows.

In the case of rank-2 Majorana mass, the Dirac mass matrix should be of the form of

$$\begin{pmatrix} \epsilon m_{ee} & \epsilon m_{e\mu} & \epsilon m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ \epsilon m_{\tau e} & \epsilon m_{\tau\mu} & \epsilon m_{\tau\tau} \end{pmatrix}, \quad (24)$$

where the dimensionless parameter ϵ is of the order of 10^{-1} and $m_{\alpha\beta} \sim 1\,\mathrm{eV}$. The non-zero elements of the Majorana mass should be of the order of 1 keV. It is important that the solar neutrino deficit can be explained by vacuum oscillations between ν_e and ν_τ, in contrast to the original framework of [9].

In the case of a rank-1 Majorana mass, the small angle MSW solution is suitable for the solar neutrino oscillations. The Dirac mass matrix should be

$$\begin{pmatrix} \epsilon m_{ee} & \epsilon m_{e\mu} & \epsilon m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ \epsilon' m_{\tau e} & \epsilon' m_{\tau\mu} & \epsilon' m_{\tau\tau} \end{pmatrix}, \quad (25)$$

where ϵ is of the order of 10^{-1} and ϵ' should satisfy the condition $(\epsilon'/\epsilon)^2 < 10^{-1}$. There is an extra oscillation mode $\Delta m^2 \sim 10^{-2}\,\mathrm{eV}^2$ or $\Delta m^2 \sim 10^2\,\mathrm{eV}^2$.

Finally, we would like to comment on the cosmological constraints in the sterile scenario. One of the constraints comes from Big Bang nucleosynthesis (BBN). The ratio of deuterium to hydrogen and the abundance of ^{4}He are determined by the ratio of neutrons to protons at the time of the weak interaction freeze out. The effective number of light neutrino flavors N_ν contributes to the energy density, which influences the expansion rate. Thus, we can obtain the upper limit of N_ν from the BBN constraint [16,17]. Although the standard BBN scenario shows $N_\nu \leq 3.6$ [16], the large lepton number asymmetry in the early universe may allow $N_\nu = 4$ [18].

In our scenario, the light sterile neutrinos contribute to the ^{4}He abundance since they have a large mixing with active neutrinos at large enough mass squared differences. Thus the increment ΔN_ν is one in the rank-2 case and two in the rank-1 case. Furthermore, the right-handed neutrinos with mass 1 keV or 100 keV also increase the effective number of neutrino species, though the increment ΔN_ν is 0.1 or so. Therefore, we need an extra mechanism which suppresses the effective number of neutrinos in the early universe[5].

A more severe problem comes from overclosing of the universe. In this paper, we conclude that the right-handed neutrino mass is about 1 keV or 100 keV. Such light neutrinos cannot decay into the standard particles. The number density of such right-handed neutrinos is at least 1/3 of the density of the other active neutrinos. Therefore, the relic right-handed neutrinos overclose our universe. We have to introduce new particles to which right-handed neutrinos can decay, or to modify our model such that the right-handed Majorana mass is heavy enough to decay into $e^+e^-\nu$ rapidly [20].

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Note added in proof: (i) New Super-Kamiokande data strongly disfavor the $\nu_\mu \to$ sterile neutrino solution at the 99% confidence level.
(ii) By Shi et al. [18] has been criticized for the assumption that a finite repopulation rate is instantaneous for temperatures above about 1.5 MeV and that the time dependence of the neutrino asymmetry is negligible [21]. Foot also computed the evolution of the number distributions taking into account these things, and gives a refutation of this criticism [22].

[5] One possibility is to consider the large uncertainty of the systematic error in the ^{4}He abundance [19]